POLARIZATION IN eRHIC ELECTRON STORAGE RING, AN ERGODIC APPROACH

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Abstract

Based on considerations of ergodicity of the dynamical system of an electron bunch at equilibrium, the preservation of polarization in an electron storage ring is estimated from the tracking of a very limited number of electrons. This has a substantial impact on required High Power Computing resources, in noticeable contrast with the method generally used that tracks tens of electron bunches, each comprised of thousands of particles, for several thousand of turns. It is also shown that a minimum number of tracking turns is required in order to ensure the numerical convergence of the linear regressions that yield depolarizing time constant values from tracking, in both methods.

INTRODUCTION

The 18 GeV eRHIC electron storage lattice used in the spin polarization simulations discussed here has been provided by S. Tepikian (BNL), detailed optical properties can be found in [1] and in references therein. The eRHIC lattice includes a double rotator system encompassing the interaction region (IR), comprised of strong solenoids and series of bends, which allows to locally move the stable spin precession direction $\vec{\imath}_{0}$, from vertical in the arcs to longitudinal at the IP. In a defect-free ring, this region of off-vertical $\vec{\imath}_{0}$ and bends is potentially a major contribution to stochastic spin diffusion.

Bunches are injected in the storage ring with both up and down polarizations, and replaced every 6 minutes in order to ensure an average polarization of about 70% over the hundreds of bunches stored [1]. A proper lattice should maintain bunch depolarization below 20% (absolute) over hundreds of bunches stored [1]. A proper lattice should include a double rotator system encompassing the interaction region (IR), comprised of strong solenoids and series of bends, which allows to locally move the stable spin precession direction $\vec{\imath}_{0}$, from vertical in the arcs to longitudinal at the IP. In a defect-free ring, this region of off-vertical $\vec{\imath}_{0}$ and bends is potentially a major contribution to stochastic spin diffusion.

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$$P(t) = P_{eq}(1 - e^{-t/\tau_{eq}}) + P_{0}e^{-t/\tau_{eq}}$$

Figure 1: A “single-particle” depolarization landscape $P(\alpha_{\gamma_{ref}})_{time=\text{int}t}$, resulting from the tracking of one particle in each one of 1024 rings with energy settings evenly distributed over $40.4 < \alpha_{\gamma_{ref}} < 40.9$ (one ring is one bin, here). Top plot: longitudinal polarization at IP after $20 \times \tau_{SR}$ (red) and after $900 \times \tau_{SR}$ (blue). Bottom plot: a zoom out on the local topology of the distribution $P(\alpha_{\gamma_{ref}})_{time=\text{int}t}$, over a reduced $40.5 < \alpha_{\gamma_{ref}} < 40.75$ interval; the topology appears to be similar at these two different timings (sort of “fractal”), and it is similar as well in any reduced interval taken in $40.45 < \alpha_{\gamma_{ref}} < 40.75$.

Essentially, the goal of tracking simulations is (i) to validate a ring design including spin matching and other compensation methods, and (ii) to determine an optimal working point $\alpha_{\gamma_{ref}}$ for that ring, amongst a polarization landscape $P(\alpha_{\gamma_{ref}})_{time=\text{int}t}$ which may typically look as displayed in Fig. 1. It results from what precedes that, given $\tau_{SP}$ (which can be estimated separately from polarization integrals proper to the lattice, and $P_{SP}$ together
with it [2]), the tracking does not need to include the self-polarization process and in that case \( P_{\text{eq}} = 0 \) and \( \tau_D = \tau_{\text{eq}} \) \((\tau_{\text{SP}} = \infty)\). \( \tau_D \) has to be long enough to ensure the required polarization survival; this eliminates, for instance, the regions surrounding \( a\gamma_{\text{ref}} = 40.4 \) or 40.85, Fig. 1-top. With \( \tau_D \gg \) tracking duration \( t \) (see next sections), one can simply use

\[
P(t)/P_0 = 1 - t/\tau_D \tag{2}
\]

A fit of this equation with the tracking data provides \( \tau_D \) and thus \( \tau_{\text{eq}} = (1/\tau_{\text{SP}} + 1/\tau_D)^{-1} \), yielding in turn \( P_{\text{eq}} = P_{\text{SP}} \times \tau_{\text{eq}}/\tau_D \) [2] and as well \( P(t = 0 \text{ min.}) \) (Eq. 1).

The simulations discussed here were performed on NERSC [3], with the tracking code Zgoubi [4]. Electron dynamics and spin diffusion in the presence of Monte Carlo SR is a long installed and, needless to say, thoroughly benchmarked feature of the code [5].

“HPC-HUNGRY”: MANY BUNCHES

In order to allow comparisons in the next section, Fig. 2 first summarizes this very HPC-hungry (High Power Computing) method, namely, many particles in a bunch, each bunch tracked for many \( \tau_{\text{SR}} \) (SR damping times) and this repeated for a number of \( a\gamma_{\text{ref}} \) values for appropriate resolution in determining the optimum ring energy setting [6].

The diffusion time constant \( \tau_D \) (the quantity of concern in this discussion) is obtained from a linear regression using Eq. 2, its value out of the regression is a function of the number of turns accounted for. It can be observed that about \( 160 \times \tau_{\text{SR}} \), 80,000 turns, are needed in the present hypotheses for \( \tau_D \) to be converged to better than 10% accuracy, Fig. 2-bottom. Converged values are, respectively \( \tau_D \approx 48, 63, 46 \) minute for \( a\gamma_{\text{ref}} = 40.615, 40.66, 40.705 \). Apparent as well from the topology in the single-particle landscape, Fig. 1, \( \tau_D \) changes quickly with \( a\gamma_{\text{ref}} \).

Note that spin tracking in these simulations only starts when the particles have reached the equilibrium regime, namely after a few damping times. Initial spins are aligned, parallel to the ideal \( \vec{u}_0 \) (longitudinal at the IP).

“ERGODIC”: SINGLE PARTICLE

The dynamical system of a high energy stored electron bunch at equilibrium is ergodic: over a long time interval, trajectories in the system cover all parts of the 6D phase space. Time averages over a trajectory are equivalent to phase space averages,

\[
\int f(\vec{X})\rho(\vec{X})d^N\vec{X} = \lim_{T\to\infty} \int_0^T f(\vec{X}(t))dt
\]

In particular, and essential in the matter of spin diffusion, over a sufficiently long time (practically, many \( \tau_{\text{SR}} \)), any electron will have explored all possible \( \delta E/E \) values within the Gaussian equilibrium momentum distribution, and a sufficiently long time that possible depolarizing effects can manifest themselves.
changes substantially during the tracking), thus the working point of concern, \(a\gamma_{\text{ref}}\), is not optimal. A contrario, observing only slow change in the spin vector means absence of harmful resonance in the energy interval that the particle spans due to SR, and potentially a viable working point.

Tracking results are shown in Fig. 3. The top plot is similar to the middle one in the HPC-Hungry case, Fig. 2, and similar \(\tau_D\) values come out of both methods. The convergence towards \(\tau_D\) is within 10\% within about 400 × \(\tau_R\) at 18 GeV, about 2.5 times the number of turns needed to converge in the multi-particle bunch technique (cf. Fig. 2-bottom).

![Figure 3](image)

Figure 3: Top plot: (x1-y1 scales) as a function of turn, average polarization of a set of single particles launched evenly spread over three different \(a\gamma_{\text{ref}}\) intervals (this is subsets of the interval displayed in Fig. 1-bottom), for 900\(\tau_{SR}\), ≈ 450,000 turns; the solid line represents Eq. 1 on a long time scale (x2-y2 scales). Bottom: convergence towards \(\tau_D\) (obtained from a linear regression using Eq. 2) as a function of the number of turns accounted for the regression, for the three different \(a\Delta\gamma_{\text{ref}}\) cases; converged values are, respectively \(\tau_D \approx 55, 49, 37\) minute, all reached within 10\% accuracy in about 200,000 turns, 400 × \(\tau_{SR}\).

Similar simulations, HPC-Hungry and “Ergodic” estimates, have been performed at 10 GeV and yield the same observations: similar topology of the polarization density in the \((S_1, a\gamma_{\text{ref}})\) space, similar diffusion time in various \(a\Delta\gamma_{\text{ref}}\) ranges, etc. [7].

**SUMMARY**

Assume similar resolution using both methods, “HPC-Hungry” and “Ergodic”, namely, the same number of reference rings, \(n\)Rings, in a given interval \(a\Delta\gamma\).

In the present hypotheses (eRHIC lattice, energy, etc.):

- first method: the HPC volume is \(n\)Rings × 10\(^3\)[particles/bunch] × 160 \(\tau_{SR}\),
- second method: the HPC volume is \(n\)Rings × 400 \(\tau_{SR}\).

This is a ratio of 10\(^3\) × 160/400 ≈ 400. Larger HPC volume translates in one or the other of, longer queues, longer computing time, more processors, greater volume of I/Os, larger data analysis HPC volume, etc.

It remains to determine how close the single-particle method can get to the accuracy of the bunch method (an ongoing work). However, it already appears an efficient first approach (in revealing fast depolarizing situation for instance), for qualifying a new lattice design, or error correction schemes. Because it is faster it allows a substantially greater reach (for instance in terms of parameter space exploration) in machine simulations and design optimizations.

**CONCLUSION**

Obviously these results are very preliminary, they are essentially indications that the HPC volume could be reduced. More simulations are required, for further inspection, comparisons between the two methods, etc. Mathematical background and support is in order.

**REFERENCES**

[1] C. Montag et al., "eRHIC design status", these Procs.