STUDY PROGRESS OF THE COUPLING RESONANCE OF THE CRAB CROSSING SCHEME IN ELECTRON-ION COLLIDER

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Abstract
Crab crossing scheme is essential collision scheme to achieve high luminosity for the future electron-ion collider (EIC). The bunch length effect of the ion beam cannot be ignored even when cooling is present compared with the wavelength of the crab cavity, therefore, the nonlinear dependence of the crabbing kick may present a challenge to the beam dynamics of the ion beam, hence an impact to the luminosity lifetime. In this paper, we present the result of numerical beam dynamics studies of the crab crossing scheme. The result indicates that there is a special coupling resonance in the nonlinear relation of the crab crossing scheme of the EIC, which dominates the luminosity degradation. And we will discuss the possible remedies for such resonance.

INTRODUCTION
The future electron ion collider (EIC) aims on achieving high luminosity. Therefore, all designs adopt crossing angles between the two beams, which allows fast beam separation, smaller beta function at interaction point (IP). The crossing angle leads to the geometric luminosity loss. The figure of merit to characterize the loss is the 'Piwinski Angle' \( \theta_p \):

\[
\theta_p = \frac{\sigma_z}{\sigma_x} \theta_c,
\]

where \( \sigma_z/\sigma_x \) are the rms longitudinal/transverse bunch size and \( \theta_c \) is the half crossing angle. To prevent the geometric beam loss we adopt the crab crossing scheme to recover the luminosity loss using crab cavities. The crab cavity exerts a kick on the beam that create a sinusoidal transverse tilt at IP

\[
x_c = \frac{\theta_c}{k_c} \sin(k_c z).
\]

In the previous studies [1], we observed that in strong-strong simulation, the luminosity degradation is a function of the frequency of the crab cavity, as shown in Fig. 1. The parameter used in the study is listed in Table 1. Such degradation is not seen in the weak-strong simulation. The indicates that degradation is associated with the cross talk of the two colliding beams.

In this paper we present the progress of studying the cause of the observed frequency dependence of the crab cavity for the eRHIC ring-ring scheme with the same parameter set as shown in Table 1 as in [1]. The other related studies related to the crab crossing is reported in [3].

NUMERICAL ERROR ESTIMATION
It is well known that the noise raised from randomness of the macro particle will contribute to the artificial change of the statistics quantities, such as the beam emittance or the luminosity, when the number of macro particle is much less than the real number of particles in the beam.

In Fig. 1, all cases shows certain luminosity degradation. We are only interested in the degradation caused by physics, instead of by noise. Figure 2 shows the luminosity degradation scaling with the number of macro particles. The top figure shows that for the ideal crab crossing case (equivalent to head-on collision), the degradation slope converge to a nearly flat line when the number of the proton macro-particle reaches two million. However the bottom figure indicates that, in the crab crossing case with crab cavity frequency 337.8 MHz, the slope of degradation obviously converges to a non-flat curve.
This gives a strong motivation to further explore the physics reason of the degradation.

**SYNCHRO-BETA RESONANCE**

We have used the strong-strong simulation code BeamBeam3D [2], which include the dynamics of both beams, to find the cause of the luminosity degradation. The number of macro-particle used in the simulations are 0.5 and 2 millions for electron and proton beam respectively. The electron beam is cut to 4 longitudinal slices and the ion is cut to 32 longitudinal slices.

We observed in [1] that no degradation is weak-strong simulation. Therefore we create a 'frozen' (rigid) electron case in BeamBeam3D by increasing of the electron energy to very high value (10e9 eV to 10e19 eV), Fig. 3 shows the comparison of luminosity degradation of the 4 cases: rigid head-on e-beam, rigid e-beam with crab crossing with 112 MHz and 336 MHz and the nominal crab crossing scheme with 336 MHz. By freezing the electron beam, the luminosity degradation decreases, which is consistent as the weak strong simulation.

The Fourier component of the ion beam horizontal centroid reveals the difference between the nominal crab-crossing scheme and the frozen e-beam scheme, as shown in Fig. 4. The common peaks of both the top and bottom figures are the peak of the ion’s betatron motion ($\mu \sim 0.32$) and the low frequency synchrotron lines ($\mu = 0.01$ and its harmonics).

These synchrotron lines appear in the horizontal centroid spectrum only because the synchro-beta coupling brought...
by the crab cavity. The difference of two spectrum is the peak around $\mu \sim 0.12$, which corresponds to the electron betatron motion imprint on the ion beam due to beam-beam interaction. This peak is missing in the frozen e-beam case since the beam-beam effect for the electron beam is negligible. From these observations, we suspect that the luminosity degradation is associated with the coupling of the proton synchrotron motion and the electron betatron motion (imprint on the ion beam).

This hypothesis can be confirmed by changing the synchrotron tunes in the strong-strong simulation. We expect that decreasing the synchrotron tune will reduce the coupling since the intensity of the high order harmonic which overlaps with the electron tune will be much smaller. The left plots in Fig. 5 shows that when the synchrotron tune decrease to 0.005 or even 0.001, the degradation is largely decreased.

Alternatively, we may increase the synchrotron tune so that the distance between synchrotron lines are wide enough to accommodate the width of the electron tune spread. The right plots in Fig. 5 shows that the synchrotron tune is increase up to 0.08. The luminosity degradation is largely improved only when the synchrotron tune is set to 0.08. As illustrated in the spectrum figure of synchrotron tune 0.08, the electron tune spread fit between the first and second harmonic of the synchrotron lines.

**SUMMARY**

We use strong-strong beam-beam simulation code to identify the cause of the luminosity degradation. The preliminary studies indicates that the numerical noise of the strong-strong beam-beam code is not the only reason that contribute to the degradation. The synchro-betatron coupling is the beam dynamics scheme which leads to the luminosity degradation. The study of varying the ion synchrotron tune endorses the judgement. However, there is not such freedom of largely changing the synchrotron tune in real machine. Therefore, further studies are required to raise a proper countermeasure to the degradation.

**REFERENCES**

