EVALUATION OF AN INTERIOR POINT METHOD SPECIALIZED IN SOLVING CONSTRAINED CONVEX OPTIMIZATION PROBLEMS FOR ORBIT CORRECTION AT THE ELECTRON STORAGE RING AT DELTA

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Abstract
The slow orbit feedback at the electron storage ring at DELTA will be upgraded with new software. Finding a set of dipole-field-strength variations which minimize the deviation of the orbit from a reference orbit requires solving a convex optimization problem subject to inequality constraints. This work focuses on exploiting properties of a special type of interior point methods, which can solve this problem, for orbit correction at DELTA. After comparing runtimes of an interior point method to a Newton-like optimization algorithm, the performance of the new slow-orbit-feedback software is assessed based on measurement results.

INTRODUCTION
DELTA is a 1.5 GeV storage-ring-based light source operated by the TU Dortmund University supplying synchrotron radiation ranging from the THz to the hard X-ray regime. The transverse orbit position of the storage ring is currently controlled via a customized SVD-based approach in service since 2005 [1]. The main goal in replacing this software is increasing reliability and performance [2]. The average orbit correction quality should be as good or better. Problems in this endeavor arise from the inequality constraints associated with the optimization problem that has to be solved to attempt an orbit correction. Other major efforts currently made to increase orbit stability and quality are a realignment of the magnet lattice, the commissioning of a fast orbit feedback [3] and an evaluation of machine-learning techniques for orbit correction [4].

ORBIT CORRECTION VIA LINEAR ORBIT RESPONSE
The miscorrection of a closed orbit can be quantified as scalar quantity

\[ \chi^2 = ||\Delta \kappa||_2^2 \]

where \( \Delta \kappa \) is a vector of deviations of the closed orbit from the reference orbit and \( ||\dots||_2 \) is the \( l_2 \) norm. Given an orbit-response matrix \( R \), an orbit correction program attempts to solve [5]

\[ \min_\theta \ ||\Delta \kappa + R \theta||_2^2 \]

to find an optimal set of deflection angles \( \theta \) which minimize \( \chi^2 \). The correction of the closed orbit is achieved by translating the vector of correction angles into a set of currents and applying these to the corrector magnets of the storage ring. A variation of this scheme is deployed at almost every storage ring worldwide.

Orbit Correction at DELTA
Adapting the introduced orbit-correction scheme for the storage ring at the DELTA facility requires the modification of the minimization problem to meet two major hardware limitations. The first of these limitations is an insufficient number of linear independent corrector magnets to correct the closed orbit at all beam position monitors (BPMs) to the reference orbit. This problem can be solved by introducing a diagonal matrix of weights \( W \) into \( \chi^2 \). A proper choice of weights increases the correction quality at important BPMs (insertion devices and septum) at the cost of orbit correction quality at all other BPMs. The second hardware limitation are the driving ranges of the corrector magnets which have to be included as inequality constraints into the optimization problem. Including weights and constraints, the modified minimization problem is

\[ \min_{\theta_{\min} \leq \theta \leq \theta_{\max}} ||W (\Delta \kappa + R \theta)||_2^2. \]

Local orbit correction possibly is an alternative to the utilization of weights [6].

NEW SOFTWARE
The new program for orbit correction [2] has matured from using the limited-memory Broyden-Fletcher-Goldfarb-Shanno algorithm with box constraints (L-BFGS-B) [7] to deploying an implementation of a primal-dual interior point method on a second-order cone from the CVXOPT Python package (qpsolve) [8] to solve the introduced, constrained minimization problem. The latter type of methods is commercially available since the 2000s and available in CVXOPT since 2009. These methods heavily exploit the problem structure leading to fast convergence for convex optimization problems and can handle arbitrary linear equality and inequality constraints [9]. Two additional properties of these methods are of special interest. The first is reliability. If an instance of the minimization problem above is solvable, the algorithm will find the optimal set of correction angles. The second property of special interest is the predictable runtime of these algorithms. They will converge for different instances of the same minimization problem within about the same amount of time and about an equal number of iterations [10]. The experimental comparison of runtimes

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(Fig. 1) displays qpcone beating L-BFGS-B by three orders of magnitude while verifying that qpcone yields better results.

**Figure 1: Runtimes of L-BFGS-B and qpcone for two sets of orbit correction steps performed on the specified dates with 106 BPMs and 112 constant inequality constraints on the corrector currents. Error bars include 95% of runtime variations. The solution of L-BFGS-B converges to the same solution as qpcone if accuracy demand on the exit condition of L-BFGS-B is increased. The RMS of the subtraction of estimated orbits of both solvers was about 0.1% in each run.**

**Software Details**

The current version of the new slow-orbit-feedback software corrects the horizontal and vertical plane separately. A coupled approach did not perform as well, yet. The orbit response matrix is regularized by cutting two or three singular values in the horizontal and three or four singular values in the vertical plane. Once $\chi^2$ drops below an adjustable limit, corrections are performed using only a single most effective corrector magnet (essentially the same as MICADO [11]). It is selected by comparing the orbit deviation $\Delta \chi$ to columns of the orbit response matrix $R$ using the Cauchy-Schwarz inequality. The reason for using only a single corrector magnet for low $\chi$ values is the resolution of the current records steering the corrector magnets. It is set to 7.0 mA, despite the bus offering a resolution of 2.4 mA [12], to suppress noise. The number of miscorrections for small $\chi^2$ values was greatly reduced this way.

It was briefly mentioned above that local orbit corrections may be an alternative to utilizing weights in order to meet orbit correction requirements in insertion devices and the injection area of the storage ring to compensate an insufficient number of correctors. All measurement results given in this article were obtained utilizing the weights concept alone. However, the ability of qpcone to handle linear equality and inequality constraints makes it possible to impose equality and inequality constraints on corrector currents and orbit positions. The current version of the new program for orbit correction allows to specify a weight and an equality or inequality constraint or no constraint at all for each BPM. Imposing constraints on BPMs has yet to be demonstrated in an experiment because it contradicts using only a single corrector for small $\chi^2$ values. The minimization problem then is infeasible due to insufficient degrees of freedom (correction angles).

**PERFORMANCE**

The new slow-orbit-feedback software was tested by correcting random perturbations applied by additional corrector magnets being part of our fast orbit feedback. Figure 2 shows a plot to assess the success rate of these corrections steps. Some of the displayed miscorrections may be attributed to hardware problems. The orbit position measured at BPM 43 in the horizontal plane (overdriven Libera Electron) and at BPM 12 in the vertical plane (broken capacitive pick-up button) were included in analysis despite their measurements being frozen. Neither of these failed correction attempts lead to significant miscorrections ($\Delta \chi > 1000$) or beam loss. The current version of the new slow-orbit-feedback software can thus be considered stable.

**Figure 2: $\chi$ value at which a correction was attempted and the resulting change $\Delta \chi$ in this value for a set of correction steps recorded in April 2018. Any correction step resulting in $\Delta \chi > 0$ is a miscorrection. Out of 219 performed correction steps, 19 or 8.7% (horizontal plane) and 34 or 15.5% (vertical plane) were miscorrections.**

**Comparison to Old Software**

The performance of the new and old slow-orbit-feedback software were compared by testing both programs on the same set of perturbations. Figure 3 displays key results of this comparison. Both softwares perform very similar in terms of average time in between correction steps and total number of steps required to correct all miscorrections (see Fig. 4). The small advantage of the new program over the old program is insignificant considering the size of the rather small test set.
The new software requires less than one second to determine an optimal set of corrector currents to perform an orbit correction step. The remaining time between steps is determined by the time required for the current sources of the corrector magnets to reach their set values and an adjustable sleep time activated after that condition is met. The sleep time was 1.0 s for the displayed measurement results. It is meant to assure that an orbit measurement for the next correction step is not perturbed by driving magnets. In consequence, there is little room for improving the mean time in between correction steps. It is mainly determined by hardware. However, none of the performed corrections used the full driving range of the correctors. All optimization problems that had to be solved during measurement were effectively unconstrained. The new software should perform better in comparison to the old software if correction steps require any correctors to use their full driving range.

![Figure 3: Performance comparison of the new slow-orbit-feedback software (orange) and the old software (blue) for a set of 20 perturbations. The Error bars of the average runtime contain 68 % of runtime variations.](image)

**CONCLUSION AND OUTLOOK**

The software component of the slow orbit feedback at the electron storage ring at DELTA is undergoing an upgrade. The minimization problem arising when attempting an orbit correction is subject to inequality constraints. The current version of the new software solves it via a primal-dual interior point method on a second-order cone from the CVXOPT Python package (qpcone). This method is fast, reliable and can handle arbitrary linear equality and inequality constraints making it possible to combine local orbit correction with inequality constraints on corrector currents and orbit positions. Local orbit correction has yet to be experimentally demonstrated. The program performance was tested on random perturbations without orbit constraints. Results indicate that the software is stable and ready for extended testing and commissioning. A comparison with the old slow-orbit-feedback software on a small and simple set of correction steps, where none of the corrector magnets used its full driving range, indicates that both programs perform equally well with the new program performing slightly better.

These results constitute a major step towards the main goal of creating a new reliable software for the slow orbit feedback of the storage ring at DELTA. The next step is commissioning the new program for user operation while conducting more tests and making adjustments. Tests should include worst-case scenarios where several correctors use their full driving range. The new software will only then display its full potential. Proposed new features include an experimental evaluation of local orbit correction and testing of different regularization techniques like Tikhonov regularization or regularization via the COBEA algorithm [13]. Live-updating the orbit response matrix based on results of correction steps may be added, as well.

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**REFERENCES**


