DETERMINATION OF RF RESONATOR AXIS INCLINATION TO 
BEAM AXIS IN ELECTRON-POSITRON STORAGE RING 

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Abstract 

We proposed and tested a method of experimental determination of the upper limit of RF resonator axis inclination to the beam axis. The electric field transverse component by such a perturbation deflects the particle trajectory. In the horizontal plane, a distortion of the closed orbit leads to a difference in the energies and spin precession frequencies of electrons and positrons in a storage collider with a single ring.

EFFECT OF PERTURBATION OF RF RESONATOR AXIS INCLINATION

Since the deflection is caused by the transverse electric field on the beam axis, the closed orbits of electrons and positrons differ from each other. Therefore, in the case of radial deflection angle in the RF resonator, there are differences in the energies (and the spin frequencies) of electrons and positrons. These differences are defined as [1]:

\[ \delta E = -2 \alpha \cdot \frac{u_0}{E} \eta x. \]  

(1)

Here \( \eta \) is the dispersion function at the resonator location, \( \alpha \) is the momentum compaction factor, and \( II \) is the machine circumference. With a \( 10^{-9} \) rad inclination angle in the equivalent resonator, which is in the center of the VEPP-4M technical section, the systematic error in the CPT invariance test [2] based on comparison of the spin frequencies of electrons and positrons [1] will be \( 7.7 \cdot 10^{-9} \) CPT against the planned \( 5 \cdot 10^{-9} \) accuracy. With the same angular perturbation for each of the five resonators in the technical section [3], the contribution to the energy shift does not exceed \( 2.5 \cdot 10^{-9} \). Due to the random distribution of the perturbations, the total contribution can be even smaller with the given spread of the angles. The energy recovery of an equivalent particle in each resonator is described by the following equation:

\[ \Delta U_i = \sin \phi \cdot U_{RF,i}. \]  

(2)

where \( U_{RF,i} \) is the voltage amplitude on the \( i \)th resonator. Then the angle of deflection equals to

\[ \alpha_i = \frac{\Delta U_i}{E}. \]  

(3)

So, the total loss per revolution is

\[ \sin \phi \cdot \sum U_{RF,i} = U_0. \]  

(4)

where \( U_{RF,i} \) is the voltage amplitude on the \( i \)th resonator. The RF frequency is modulated as follows: \( \omega = \omega_{RF} \Delta \omega \cdot \cos(\omega t) \).

DESCRIPTION OF METHOD

The proposed method consists in resonant excitation of betatron oscillations by modulating the frequency of the RF system master oscillator (see Fig. 1). There are five RF resonators (R2, R3, R4, R5, and R6) operating in the main mode \( E_{010} \) in the technical section of VEPP-4M. Parameters of the resonators are given in Table 1.

<table>
<thead>
<tr>
<th>Resonator</th>
<th>Q</th>
<th>U</th>
<th>fRF (MHz)</th>
<th>KM</th>
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</thead>
<tbody>
<tr>
<td>R2</td>
<td>48500</td>
<td>11</td>
<td>181.83</td>
<td>0.0052</td>
</tr>
<tr>
<td>R3</td>
<td>46100</td>
<td>246</td>
<td>181.76</td>
<td>0.0055</td>
</tr>
<tr>
<td>R4</td>
<td>47600</td>
<td>266</td>
<td>181.73</td>
<td>0.0053</td>
</tr>
<tr>
<td>R5</td>
<td>48500</td>
<td>7</td>
<td>181.76</td>
<td>0.0052</td>
</tr>
<tr>
<td>R6</td>
<td>43500</td>
<td>8</td>
<td>181.76</td>
<td>0.0058</td>
</tr>
</tbody>
</table>

Figure 1: Scheme of the experiment.

The resonator oscillation spectrum has three main frequencies: \( \omega_{RF} \) and \( \omega_{RF} \pm \omega_\delta \). A combination of two side band frequencies produces oscillations of the form \( \sin(\omega_{RF}) \cdot \sin(\omega t) \). All the resonators act in-phase on an equilibrium phase, and therefore \( \sin(\omega_{RF}) \) can be considered as a constant. Thus, it is possible to use phase modulation (PM) to build up transverse oscillations at resonant frequencies \( \omega_\delta = f_0 \cdot \{ 1 - \{ v_{c3} \} \} \), where \( \{ \} \) is the non-integer part of the betatron frequency in the X and Z directions, and \( f_0 \) is the revolution frequency.

kicker

scintillation counters of Touschek polarimeter

βK, χK

RF resonator

PM (fM ~ 300 – 400 kHz)
Monitoring of the betatron resonance was carried out with the help of two scintillation counters of the Touschek polarimeter of the VEPP-4M collider [3]. The intensity of scattering of Touschek particles is inversely proportional to the beam volume. Therefore, at resonance the load of the counters falls down due to increase respective in the transverse size of the beam.

**Excitation of Betatron Oscillations by Phase Modulation of RF System**

Three series of experiments were carried out, the first two involving the RF resonators. The frequency of the external generator, that sets the phase modulation, was scanned in the range of 372.5–379.5 kHz for the X resonance (radial oscillations) and of 350.5-357.5 kHz for the Z resonance (vertical oscillations). Most of the experiments were conducted with resonators R3 and R4 tuned and the rest resonators untuned. It is obvious that the main perturbation is produced by the tuned resonators with a large field amplitude (4).

![Figure 2](image_url)  
**Figure 2:** Typical behavior of the Touschek counters over time during scanning the PM frequency near the vertical betatron resonance.

![Figure 3](image_url)  
**Figure 3:** Scan with the electrostatic separation plates turned on.

Figure 2 shows the typical behavior of the counters load near the Z resonance in scanning the RF generator modulation frequency. The resonance FWHM \( \frac{\Delta v_z}{v_z} \approx 10^{-3} \) in relative units.

In the second series of experiments, the Z oscillations were excited at a frequency of 342 kHz. We studied the influence of the driving transverse force (the magnetic component in the E010 mode) on the beam varying the vertical displacement of the closed orbit in the resonators.

The first three experiments were performed with the electrostatic separation plates turned on. In this case, in addition to the effect of the resonator axis inclination, the particles are under the action of a magnetic force, which is proportional to the orbit displacement. Experiments with the plates turned off showed the resonance peak to decrease more than two-fold (Figs. 3 and 4).

![Figure 4](image_url)  
**Figure 4:** Scan of the PM frequency with the electrostatic separating plates turned off and a vertical displacement of the closed orbit down of 0.82 mm due to the TZ bump.

All the experiments were carried out on the same PM control signal span. So, we can assume that the value of the effect under study is within 5%.

**Calibration Experiments with Kicker**

The third series of experiments is related to calibration. The external generator was connected to the depolarizing kicker vertical plates. It was necessary to determine the threshold for the transverse electric force in the resonators. This threshold was found from the agreement of the resonance effect in the calibration series with the data of the PM experiments with minimal influence of the orbit transverse displacement. To do this, the vertical electrostatic separation of orbit in the technical section was turned off. The kicker excited the Z resonance at a frequency of 342 kHz. Thus, the calibration is related to evaluation of the vertical angle of the resonator axis inclination. Figures 5 and 6 show typical dependences of the behavior of the load of the counters for large and small voltages applied to the kicker.

A significant change in the resonance particle scattering rate, which was approximately 5%. This observation may indicate presence of a stronger factor, which we associate with the resonator axis inclination to the beam axis in the vertical plane.
where $H_{\text{eff}}$ is defined through the amplitude of the voltage across the plates, and $HR$ is the magnetic rigidity at the energy of the experiments of 1.85 GeV. From the calculations we have $\chi_K = 3.2 \cdot 10^{-9} \text{ rad.}$

The voltage across the resonator for an equilibrium particle with a phase $\phi_0$ is written as

$$U_{RF} \sin \phi_0 + 2U_{RF} \cos \phi_0 \cdot J_1(\Delta \phi_m) \cos \omega t. \quad (6)$$

$J_1(\Delta \phi_m)$ is the first-order Bessel function. The second term of this sum corresponds to the driving force. Then the angle of deflection in the resonator is determined from the following expression:

$$\chi_{RF} = 2\alpha_z e^{U_{RF} \cos \phi_0 J_1(\Delta \phi_m) \Delta \phi_m} \quad (7)$$

We find the average vertical angle of axis inclination for the effective RF resonator ($R_3 + R_4$) with $U_{RF} = 440 \text{ kV}$ and $K_M = 0.0054$. The relationship between the deflection angles in the kicker $\chi_K$ and in the resonator $\chi_{RF}$ is given by the following expression:

$$\chi_K = \chi_{RF} \sqrt{\frac{\bar{\beta}_{Z(RF)}}{\bar{\beta}_{Z(K)}}} \quad (9)$$

where $\bar{\beta}_{Z(RF)} = \sqrt{\bar{\beta}_{K}^2 + \bar{\beta}_{K}^2 + 2\bar{\beta}_{K} \bar{\beta}_{K} \cos \Delta \Psi_z}$ and $\Delta \Psi_z = 2.348 \text{ rad}$ is the betatron phase incursion between resonators $R_3$ and $R_4$. From this equality we obtain that $\chi_{RF} = 3.7 \cdot 10^{-9} \text{ rad.}$ Then the vertical angle of inclination of the resonator axis found from formula (7) is $\alpha_z \leq 0.01 \text{ rad.}$

**CONCLUSION**

One can suppose that similar angles of inclination may occur in the median plane. Hence, the estimated contribution of the RF resonator axis angle perturbation to the error of the CPT test experiment will not exceed $10^{-8}$ for resonator $R_5$. At the same time, this value is $<10^{-9}$ for resonator $R_6$, due to the small dispersion function value at its location. Thus, it is desirable to perform the CPT test using resonator $R_6$.

More precise determination of the contribution of the resonators to the CPT test experiment error requires conduction of a calibration experiment on radial oscillations using a kicker with the radial plates. Such variant, in principle, is possible but needs special technical preparation.

**REFERENCES**

