BEAM-BEAM STUDIES FOR SUPER PROTON-PROTON COLLIDER

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SPPC

**SPPC design**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>C 100 km</td>
</tr>
<tr>
<td>Beam energy</td>
<td>E 37.5 GeV</td>
</tr>
<tr>
<td>Normalized Emittance</td>
<td>γε 2.4 μm</td>
</tr>
<tr>
<td>beta function at IP</td>
<td>b* 0.75 m</td>
</tr>
<tr>
<td>Bunch population</td>
<td>Np 1.5x10^{11}</td>
</tr>
<tr>
<td>Full crossing angle</td>
<td>θc 110 μrad</td>
</tr>
<tr>
<td>Number of IP</td>
<td>NIP 2</td>
</tr>
<tr>
<td>Number of bunch</td>
<td>Nb 10,080</td>
</tr>
<tr>
<td>Pea Luminosity</td>
<td>L 1.01x10^{35} cm^{-2}s^{-1}</td>
</tr>
</tbody>
</table>
X-separation

82 long range/IP

Horizontal separation (sigma)

head-on

Long range interaction: every 3.75m.
Study of the beam-beam effects in SPPC

• Beam-beam simulation using weak-strong model
  • HH/HV crossing without long range
  • HH/HV crossing with long rage

• Resonances caused by the beam-beam interactions
  • HH crossing (single collision) with long rage
  • HV crossing with long range
Weak-strong simulation

• Round beam collision

\[ \Delta p_r(s_i) = \frac{2N_{p,i} r_p}{\gamma} \frac{1}{r} \left[ 1 - \exp \left( -\frac{r^2}{2\sigma_r(s_i)^2} \right) \right] \]

\[ \Delta p_z(s_i) = \frac{N_{p,i} r_p}{\gamma} \frac{1}{\sigma_r(s_i)^2} \exp \left( -\frac{r^2}{2\sigma_r(s_i)^2} \right) \frac{d\sigma_r^2(s_i)}{dz}. \]

• Integrate along bunch length, Long range, assume round beam

\[ M(js^*) = \prod_{i=0}^{N-1} e^{-U_{bb}(x,s_i)} M(s_i, s_{i+1}) \]

\[ = \begin{bmatrix} \prod_{i=0}^{N-1} M^{-1}(s_i, s^*) e^{-U_{bb}(x,s_i)} M(s_i, s^*) \end{bmatrix} M(s^*) \]

Operated left to right

Transfer IP to LR (long range interaction point)  bb  LR to IP  Revolution matrix  Simplified model
Simulation results

- \( N_p = 10^5, 10^6 \) turns
- Fit luminosity decay rate, emittance growth rate

Luminosity decreases

Beam size increases
Luminosity decrement, beam lifetime in the simulation

### HH crossing, w or wo crossing angle

<table>
<thead>
<tr>
<th>Np(10^{11})</th>
<th>x/IP</th>
<th>dL/L0 (0mrad)</th>
<th>dL/L0 (110mrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.007633</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.0</td>
<td>0.015266</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7.5</td>
<td>0.038165</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>0.076330</td>
<td>-0.26</td>
<td>-6.66</td>
</tr>
<tr>
<td>30</td>
<td>0.152660</td>
<td>-2.36</td>
<td>-210</td>
</tr>
<tr>
<td>75</td>
<td>0.381650</td>
<td>-380</td>
<td>-494</td>
</tr>
<tr>
<td>150</td>
<td>0.763300</td>
<td>-552</td>
<td>-436</td>
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</table>

### HV crossing

<table>
<thead>
<tr>
<th>Np(10^{11})</th>
<th>x/IP</th>
<th>dL/L0 (per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.007633</td>
<td>0</td>
</tr>
<tr>
<td>3.0</td>
<td>0.015266</td>
<td>0</td>
</tr>
<tr>
<td>7.5</td>
<td>0.038165</td>
<td>-14.5</td>
</tr>
<tr>
<td>15</td>
<td>0.076330</td>
<td>-203</td>
</tr>
<tr>
<td>30</td>
<td>0.152660</td>
<td>-309</td>
</tr>
<tr>
<td>75</td>
<td>0.381650</td>
<td>-231</td>
</tr>
<tr>
<td>150</td>
<td>0.763300</td>
<td>-215</td>
</tr>
</tbody>
</table>

### HH crossing, with long range bb interactions.

<table>
<thead>
<tr>
<th>HH cross Np(10^{11})</th>
<th>Beam life [h] 7s</th>
<th>Beam life [h] 5s</th>
<th>dL/L0</th>
<th>HV cross Beam life [h] 7s</th>
<th>Beam life [h] 5s</th>
<th>dL/L0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.0153</td>
<td>No lost</td>
<td>148</td>
<td>No lost</td>
<td>221</td>
<td>0</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0305</td>
<td>No lost</td>
<td>21.5</td>
<td>-0.12</td>
<td>264</td>
<td>17.5</td>
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<tr>
<td>4.5</td>
<td>0.0457</td>
<td>2.14</td>
<td>1.35</td>
<td>-2.26</td>
<td>8.76</td>
<td>3.15</td>
</tr>
</tbody>
</table>

Red is disaster.
Resonances induced by the beam-beam interactions

• Beam-beam force (round beam)

\[ F_r(r) = 2C_p \frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r}, \quad C_p = \frac{N_p r_p}{\gamma} \]

• Beam-beam potential

\[ U_{bb}(r) = -\int_r^\infty F(r')dr' = -2C_p \int_r^\infty \frac{1 - e^{-\frac{r'^2}{2\sigma^2}}}{r'}dr' \]

• Integral over bunch length, long range, multi-IP

\[ \mathcal{M}(s^*) = \prod_{i=0}^{N-1} e^{-U_{bb}(x, s_i)} M(s_i, s_{i+1}) \]

\[ = \left[ \prod_{i=0}^{N-1} M^{-1}(s_i, s^*)e^{-U_{bb}(x, s_i)} M(s_i, s^*) \right] M(s^*) \]

\[ = \left[ e^{-\int U_{bb}(M(s^*, s')x, s')ds'} \right] M(s) \]
One turn map, resonance driving term for beam-beam interactions

- Integrated beam-beam interaction
  \[ \int U_{bb}(M(s^*, s')x, s')ds' \Rightarrow U_{bb} \]

- Hamiltonian describing one turn map
  \[ H = \mu \cdot J + \delta_P(s)U_{bb} \]

- Fourier expansion of Hamiltonian, resonance driving term
  \[ U_m = \frac{1}{(2\pi)^2} \int d\phi_x d\phi_y U_{bb} e^{im\phi} \]

- Analytic integration for \( \phi_{x,y} \) is complex for a long range and crossing collisions.

- Direct integration for \( \phi_{x,y} \) is used now.
Integration for $\phi_{xy}$

• Choose $\Delta \phi_{xy} = 2\pi/100 - 2\pi/400$

$$U_m = \frac{1}{(2\pi)^2} \int d\phi_x d\phi_y U_{bb} e^{i m \phi}$$

$$\frac{\partial U_0}{\partial J_x} = \frac{1}{(2\pi)^2} \int d\phi_x d\phi_y \frac{\partial U_{bb}(r)}{\partial x} \frac{\partial x}{\partial J_x}$$

$$= -\frac{1}{(2\pi)^2} \int d\phi \frac{x}{2J_x} F_x(x - x_{LR}, y - y_{RL})$$

$$\frac{\partial U_0}{\partial J_y} = -\frac{1}{(2\pi)^2} \int d\phi \frac{y}{2J_y} F_y(x - x_{LR}, y - y_{RL})$$

$$F_x(x, y) = 2C_p \left( 1 - e^{-\frac{x^2}{2r^2}} \right)$$

$$2\pi \Delta \nu_x = \frac{\partial U_0}{\partial J_x} \quad 2\pi \Delta \nu_y = \frac{\partial U_0}{\partial J_y}$$

$$\frac{\partial^2 U_0}{\partial J_x^2} = \frac{1}{(2\pi)^2} \int d\phi$$

$$\left[ \frac{1}{2} \sqrt{\frac{\beta_x}{2J_x}} \cos(\phi_x + \phi_x) F_x(x - x_{LR}, y - y_{RL}) \right]$$

$$\beta_x \cos^2(\phi_x + \phi_x) \frac{\partial F_x}{\partial x}$$

$$\frac{\partial^2 U_0}{\partial J_y^2} = -\frac{1}{(2\pi)^2} \int d\phi$$

$$\sqrt{\frac{\beta_x \beta_y}{4J_x J_y}} \cos(\phi_x + \phi_x) \cos(\phi_y + \phi_y) \frac{\partial F_x}{\partial y}$$

$$\beta_y \cos^2(\phi_y + \phi_y) \frac{\partial F_x}{\partial y}$$

$$\frac{\partial^2 U_0}{\partial J_i \partial J_j} = \frac{1}{(2\pi)^2} \int d\phi$$

$$\left[ \frac{1}{2} \sqrt{\frac{\beta_y}{2J_y}} \cos(\phi_y + \phi_y) F_y(x - x_{LR}, y - y_{RL}) \right]$$

$$\beta_y \cos^2(\phi_y + \phi_y) \frac{\partial F_y}{\partial y}$$

$$2\pi \frac{\partial \nu_i}{\partial J_i} = 2\pi \frac{\partial \nu_j}{\partial J_j} = \frac{\partial^2 U_0}{\partial J_i \partial J_j}$$
Integration (summation) along s

\[ \int U_{bb}(M(s^*, s'), x, s')ds' \]

- Integrate along s with taking account of bunch length and long range

\[ U_m(J, z) = \frac{1}{(2\pi)^2} \int \lambda_p(z')ds \int d\phi_x d\phi_y e^{im\phi} U_{bb}(r) \]

\[ s = (z - z')/2 \]
\[ \lambda_p(z') = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z'^2}{2\sigma_z^2}\right) \]

\[ U_m = \frac{1}{(2\pi)^2} \sum_{LR} d\phi_x d\phi_y U_{LR} e^{im \cdot \phi} \]

- Tune shift and its slope (2\textsuperscript{nd} derivatives) are calculated by \( U_0 \).
Tune shift (direct integral for Ud$\phi$)

Each line is drawn for given $J_x$, $0-32\varepsilon_x$. 
Uxx, Uxy, Uyy
Resonances

- A resonance occurs when the betatron amplitude satisfying
  \[ m_x \nu_x (J_R) + m_y \nu_y (J_R) = n \]

- The resonance line crosses the tune spread area.

- Resonance base
  \[ P_1 = \frac{J_x - J_{x,R}}{m_x} \]
  \[ \psi_1 = m_x \phi_x + m_y \phi_y \]
  - Fixed point \( P_1 = 0 \).
  - For larger \( P_1 (J_x) \), \( \psi_1 \) moves due to detuning from the resonance condition.
  - Motion in \( P_1 (J_x \text{ around } J_{x,R}) \), \( \psi_1 \) space depicts separatrix.
  - Keep \( P_2 = (J_x - J_{x,R})/m_x + (J_y - J_{y,R})/m_y \)

Each line is drawn for given \( J_x, 0-32 \varepsilon_x \).
Betatron amplitude satisfying resonance conditions

• The beam-beam force is symmetric for $y$ in the horizontal crossing.

• Only even $m_y$ appears $(4, -1) \Rightarrow (8, -2)$. 
Resonance width

- Characterize emittance growth

\[ P_1 = \frac{J_x - J_{x,R}}{m_x} \quad \psi_1 = m_x \phi_x + m_y \phi_y \]

\[ H = \frac{\Lambda}{2} P_1^2 + U m (J_R) \cos \psi_1. \]

\[ \Lambda = m_x^2 \frac{\partial^2 U_{00}}{\partial J_x^2} + 2m_x m_y \frac{\partial^2 U_{00}}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U_{00}}{\partial J_y^2} \]

\[ \Delta P_1 = 2 \sqrt{\frac{U m}{\Lambda}} \quad \Delta J_x = 2m_x \sqrt{\frac{U m}{\Lambda}}. \]

Half width
Standard map

- Transfer (revolution) map for H
  \[ H = \frac{\Lambda}{2} P_1^2 + U_m (J_R) \cos \psi_1. \]
  \[ I = \Lambda P_1, \theta = \psi_1, t = s/L \]

- Standard map
  \[ K = \Lambda U_m \]
  \[ I_{t+1} = I_t + K \sin \theta_t \]
  \[ \theta_{t+1} = \theta_t + I_{t+1} \]

- Resonance half width
  \[ \Delta I = 2\sqrt{K} \]
  \[ \Delta P_1 = 2\sqrt{\frac{U_m}{\Lambda}} \]
  \[ \Delta J_x = 2m_c \sqrt{\frac{U_m}{\Lambda}}. \]

- K, which is called “stochasticity parameter” is very small, \( K < 10^{-4} \).
  Strong chaotic system is \( K > 1 \).

J-PARC Space charge \( K \sim 0.01 \)
Resonance width in amplitude space

- SPPC, Np=1.5x10^{11}, 1-IP H crossing

FMA analysis
Resonance width in amplitude space

- SPPC, $N_p=3 \times 10^{11}$, 2-IP, $(\nu_x, \nu_y)=(0.29,1.30)$
- 7-th order resonances, Studied numerically in K. Ohmi, F. Zimmermann, PRST-AB18,121003 (2015)
- The width is larger for finite $z$. Odd order resonances can be enhanced by finite $z$. 

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![Graph showing resonance widths](image-url)
Synchrotron motion

- The beam-beam potential is calculated as a function of $z$.
- Tune shift dependence on $z$

**Horizontal crossing**

$$z = \sqrt{\frac{2\beta_z J_z}{\epsilon} \cos \phi_z}$$

$$\delta = \sqrt{2J_z/\beta_z \sin \phi_z}$$

**Horizontal/Vertical crossing**
Beam-beam potential

• Synchrotron motion is very slow compared with betatron motion.

• $U$ is separated to average, synchrotron, synchro-betatron terms

$$U_{bb} = U_{O,0} + \sum_{m_z \neq 0} U_{O,m_z} e^{-im_z \phi_z} + \sum_{m \neq 0, m_z} U_{m,m_z} e^{-im \cdot \vec{\phi} - im_z \phi_z}$$

$$z = \sqrt{2 \beta_z J_z} \cos \phi_z$$
$$\delta = \sqrt{2 J_z / \beta_z} \sin \phi_z$$

• Fourier component for the synchrotron motion

$$U_{m,m_z}(J,J_z) = \frac{1}{2\pi} \int U_m(J,z) e^{im_z \phi_z} d\phi_z$$

$$U_m(J,z) = \int \lambda_p(z') ds \int \frac{d\phi}{(2\pi)^2} e^{im\phi} U_{bb}(r,z)$$

• Resonance condition for synchro-beta resonances

$$m_x \nu_x(J,J_z) + m_y \nu_y(J,J_z) + m_z \nu_z = n$$
Resonance width for the synchro-betatron resonances

- Resonance with \((m,0)\) and its sideband \((m,m_z)\)

\[
\tilde{U}(J, J_z) = U_{0,0}(J, J_z) + \sum_{m \neq 0, m_z} U_{m,0}(J, J_z) e^{-im \cdot \phi - im_z \phi_z}
\]

\[
\tilde{H} = \tilde{U} = \frac{\Lambda m}{2} P_1^2 + U_{m,m_z}(J_R, J_z) \cos \psi_1
\]

\[
\Lambda m = m_x^2 \frac{\partial^2 U_{0,0}}{\partial J_x^2} + 2m_x m_y \frac{\partial^2 U_{0,0}}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U_{0,0}}{\partial J_y^2}
\]

- Separation of the sideband

\[
\delta P_1 = \frac{\mu_z}{\Lambda m}
\]

- Overlapping condition

\[
\Delta P_1 > \frac{2}{3} \delta P_1 \quad 3\sqrt{\Lambda m U_{m,m_z}} > 2\mu_z
\]

- Condition: The resonance width is larger than their separation.
Modulation due to the synchrotron motion

- Synchrotron motion should be considered even if the resonance condition is not satisfied, because it is very slow.

\[
\hat{U}(J, J_z, t) = \sum_{m_z \neq 0} U_{0,m_z} e^{-im_z \mu_z t}
\]

- Standard map for a synchro-beta resonance with the modulation

\[
\begin{align*}
\dot{I}_{t+1} &= I_t + K m_{m_z} \sin \psi_1 \\
\dot{\theta}_{t+1} &= \theta_t + I_{t+1} + \sum_{m_z \neq 0} \frac{\partial U_{0,m_z}}{\partial J} \cdot m \cos(m_z \mu_z t)
\end{align*}
\]

- Chaotic area due to the modulation

\[
\Delta P_1 = \max_{m_z} \left( \frac{1}{\Lambda} \frac{\partial U_{0,m_z}}{\partial J} \cdot m \right)
\]
No overlap or weak modulation diffusion

- SPPC, $N_p = 1.5 \times 10^{11}$, 1-IP, $\Delta m_z = 2, \mu_z \rightarrow 2 \mu_z$
- $K^{1/2} < 4 \mu_z / 3 = 0.008$ no overlap between synchrotron sidebands
- $m.dU/dJ << K^{1/2}$ stochastic area is narrower than the resonance width.
Higher intensity

• SPPC, Np=3x10^{11}, 2-IP, (v_x, v_y)=(0.29,1.30), (7,0) resonance

• Resonances m_z=0 and 2 can overlap. \( \Delta m_z=2, \mu_z \rightarrow 2\mu_z \)

• \( m.dU/dJ<K^{1/2} \) stochastic area is narrower than the resonance width, but contributes overlap of sidebands with \( m_z=2 \).

• Many resonances overlap (7,0),(6,1)...(0,7) and their 2^{nd} synchrotron sidebands. It is disaster in this parameter.
Chromaticity

- Hamiltonian

\[ H = \mu_0 \cdot J + 2\pi \delta \xi \cdot J + \delta_P(s)U_{bb} \]

- Modulation term

\[ U_\xi = 2\pi \xi \cdot mP_1 \sqrt{2J/\beta_z} \sin \mu_s t \]

- Stochastic area due to chromaticity

\[ \Delta P_1 = \frac{2\pi \sigma_\delta \xi \cdot m}{\Lambda m} \sqrt{\frac{2J}{\varepsilon_z}} \]
Summary

• Incoherent beam-beam effects have been studied for SPPC.
• Beam-beam simulation using weak-strong model showed the beam-beam limit between $\xi=0.03-0.045$, where 82 long range interactions were considered.
• Nonlinear resonances which cause the emittance growth have been studied considering head-on and long range interaction.
• Weak resonance effect in the design parameter at $(\nu_x, \nu_y)=(0.31,1.32)$.
• Synchrotron sideband effect was seen, but not very strong. The long range interaction is independent of $z$.
• Higher intensity than the design and tune near a strong resonance showed disaster, as is consistent with the weak-strong simulation.
Thank you for your attention